Some Tractable Instances of Interval Data Minmax Regret Problems

Bounded Distance From Triviality

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Robust optimization

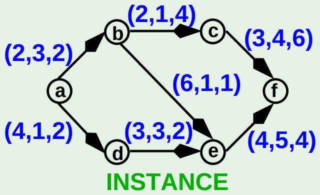
Robust graph optimization problems [Kouvelis and Yu, 1997]

Problems where the weights are ill-known due to uncertainty or imprecision. A set of scenarios is defined, with one scenario for each possible assignment of weights to the graph.

Two approaches according to the way the scenarios are defined:

1. the discrete scenario model where each weight is a vector, every component of which is a particular scenario;

A robust shortest path problem in the discrete scenario model



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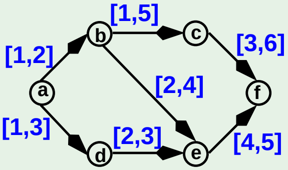
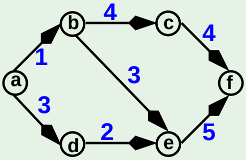
Two approaches according to the way the scenarios are defined:

2. the interval model where each weight is an interval and the set of scenarios is defined implicitly as the Cartesian product of all the intervals.

A robust shortest path problem in the interval model





Robustness measures

In robust optimization, it is assumed that:

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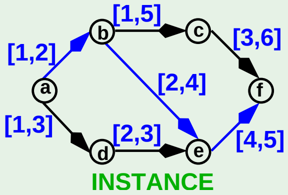
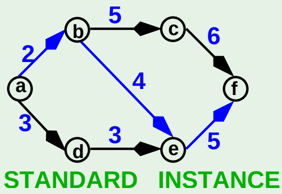
Two main robustness measures of solutions are studied [Kouvelis and Yu, 1997]:

- the maximum weight of a solution over all scenarios,
- the maximum regret of a solution.

Algorithm for the maximum weight measure





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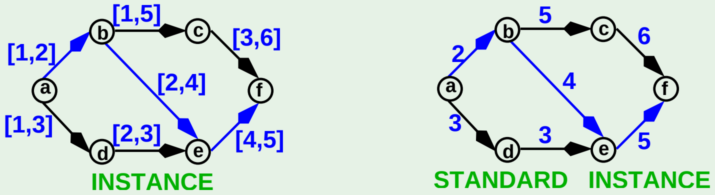


An optimal solution is path (a,d,e,f) with max weight 11.

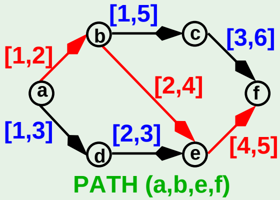
Maximum regret of a solution

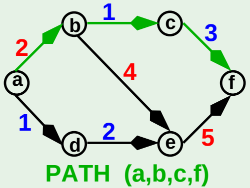
The max regret measure consists of evaluating a solution on the basis of its maximal deviation from the optimal value over all scenarios. The scenario for which the max regret occurs is called worst case scenario.

Theorem (Averbakh, 2001)

For a minimisation problem, the worst case scenario for a solution is the one where the chosen edges are set to the maximum possible weight and the unchosen ones are set to the minimum possible weight.

Maximum regret of a path





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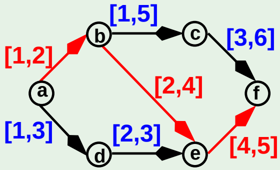
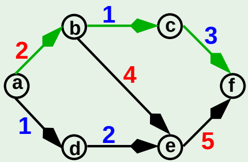


PATH (a,b,e,f)



PATH (a,b,c,f)

The max regret of (optimal) path (a,b,e,f) is $(2+4+5)-(2+1+3)=5$.

Our work

The robust shortest path (spanning tree) problem with the max regret criterion is strongly NP-hard [Averbakh and Lebedev, 2004].

In this talk

We identify tractable instances of the robust shortest path problem and the robust minimum spanning tree problem.

Distance from triviality [Guo et al., 2004]

One introduces parameters that measure the distance from well known solvable instances, and one shows that the instances remain tractable as long as the parameter is bounded by a constant.

Upper bounded number of non degenerate intervals

Trivial instance: all the intervals reduce to a single point (degenerate).

Distance: number k of non degenerate intervals.

Result: if k is bounded by constant, problem polynomially solvable by a brute force algorithm [Averbakh and Lebedev, 2004].

Upper bounded minmax regret

Upper bounded minmax regret

Trivial instance: instance the minmax regret of which is zero.

Distance: the value k of the minmax regret.

We now show to solve a trivial instance.

Theorem (Kasperski and Zielinski, 2006)

By computing an optimal solution S of the (standard) instance weighted by the middle of every interval, one obtains a 2-approximation of the minmax regret $R(S^*)$: $R(S) \leq 2R(S^*)$.

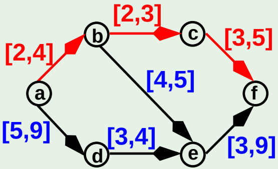
Consequently, if the minmax regret is zero, then the solution computed this way is a minmax regret solution !

Proof. Since $R(S) \leq 2R(S^*)$ and $R(S) \geq R(S^*)$, we have:

$$R(S^*) = 0 \iff R(S) = 0.$$

From minmax regret 1 to minmax regret 0 (1/2)

Minmax regret 1



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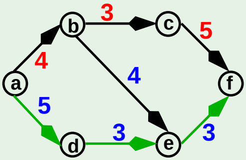

PATH (a,d,e,f)

From minmax regret 1 to minmax regret 0 (1/2)

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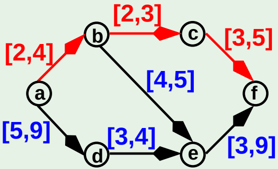

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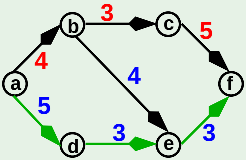
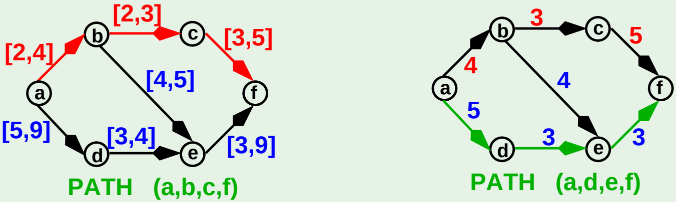


PATH (a,d,e,f)

The optimal path is path (a,b,c,f) with max regret $(4+3+5)-(5+3+3)=1$.

From minmax regret 1 to minmax regret 0 (1/2)

Minmax regret 1



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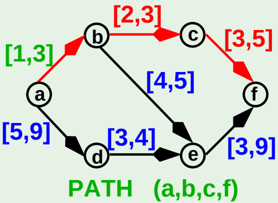
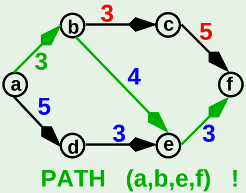
Idea

Reduce by 1 the interval of an edge of a minmax regret path (by trying each edge) $\Rightarrow$ get an instance with minmax regret zero.

From minmax regret 1 to minmax regret 0 (2/2)

Minmax regret 0 ?





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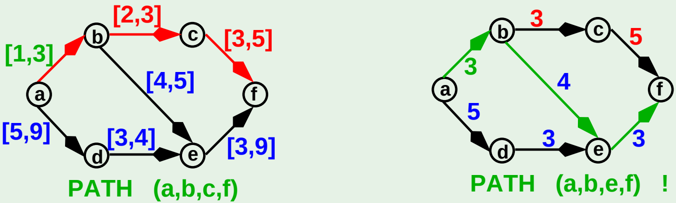

Problem: edge (a,b) belongs to a shortest path in the worst case scenario ⇒ the regret is still 1.

From minmax regret 1 to minmax regret 0 (2/2)

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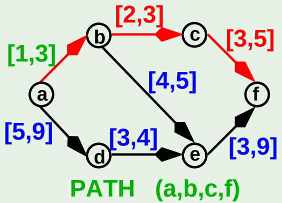

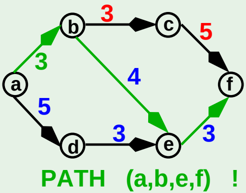

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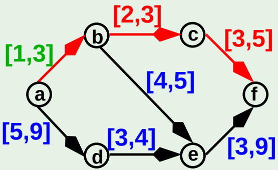
Key lemma

In any minmax regret path, there exists at least one edge that does not belong to any shortest path in the worst case scenario.

We need to test *m* instances to find an optimum solution of regret 1 if it exists.

From minmax regret 1 to minmax regret 0 (2/2)

Minmax regret 0 ?



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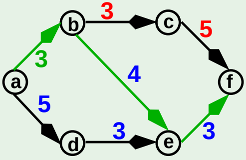

PATH (a,b,e,f) !

Problem: edge (a,b) belongs to a shortest path in the worst case scenario $\Rightarrow$ the regret is still 1.

Key lemma

In any minmax regret path, there exists at least one edge that does not belong to any shortest path in the worst case scenario.

We need to test m instances to find an optimum solution of regret 1 if it exists.

More generally: we need to test m^k instances to find an optimum solution of regret k if it exists.

Other results

Upper bounded number of interval intersections (spanning tree)

Trivial instance: intervals are pairwise disjoint.

Distance: number k of intervals intersecting at least one other interval.

Result: if k is bounded by a constant, problem polynomially solvable.

Upper bounded reduction complexity (shortest path)

Trivial instance: series-parallel graphs (proved pseudo-polynomial by [Kasperski and Zielinski, 2006]).

Distance: value k of reduction complexity.

Result: if k is bounded by a constant, problem pseudo-polynomially solvable.

Upper bounded treewidth and max degree (shortest path)

Trivial instance: trees, cycles and chains.

Distance: treewidth k and max degree Δ.

Result: if k and Δ are bounded by constants, problem pseudo-polynomially solvable.

Conclusion

- We have presented tractable instances of interval data minmax regret problems (shortest path and minimum spanning tree).
- We conjecture that RSP can be pseudopolynomially solved in graphs with bounded treewidth (without any degree restriction).
- The issue we investigate here can also be investigated in the discrete scenario model.

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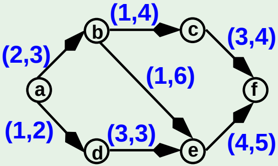
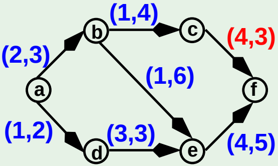
A robust shortest path problem in the discrete scenario model



Polynomial on this type of instance



NP-hard on this type of instance !

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