Untangling a planar graph

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Outline

Statement of the problem and previous work

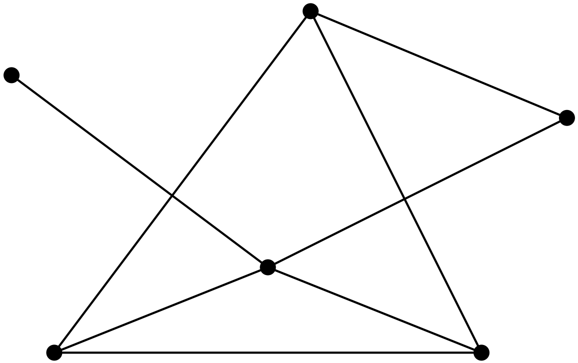
The basic idea for our lower bound construction

Our results

Concluding remarks

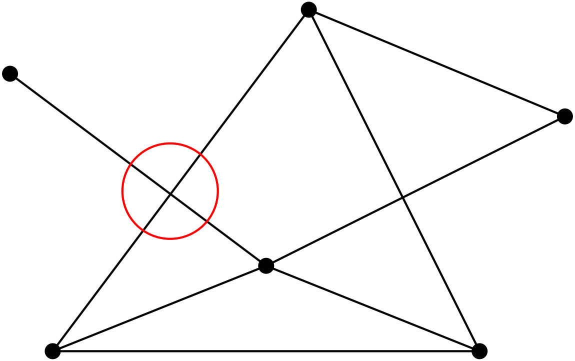
Geometric graphs

A graph $G = (V, E)$ with a fixed straight line drawing δ in the plane.



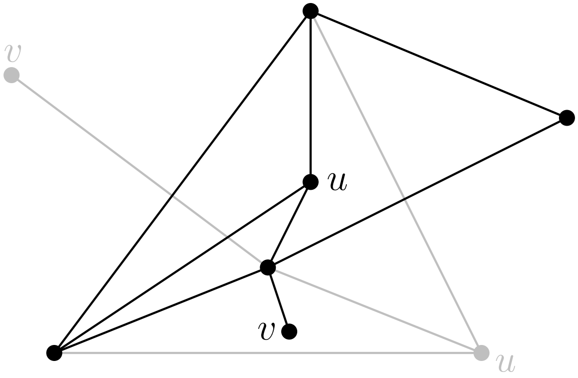
Crossing edges

Two edges that share a point that is not an endpoint of both.



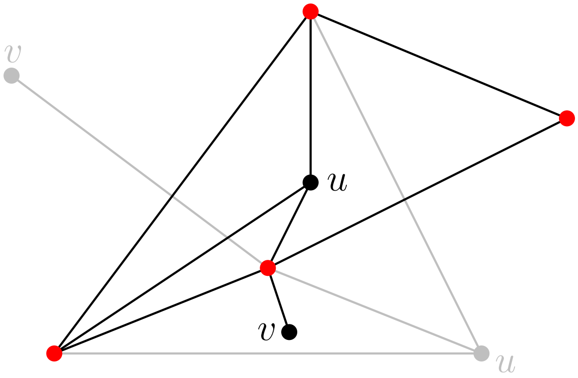
Untangling a geometric graph

Move vertices to new positions to get rid of all crossing edges.



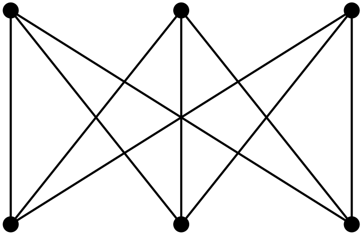
Fixed vertices

Vertices that are not moved during the untangling process are called *fixed*.



Restriction to planar graphs

Clearly, not every geometric graph can be untangled.



So, we assume that G is planar, that is, *there exists* a drawing without crossing edges.

Statement of the problem

- Given a straight line drawing δ of a planar graph G we define
$$\mathrm{fix}(G, \delta)$$
as the maximum number of vertices that can be kept fixed when untangling δ.
- Given a planar graph G we define
$$\mathrm{fix}(G)$$
as the minimum of $\mathrm{fix}(G, \delta)$ over all possible straight line drawings δ of G.

Statement of the problem

- **Goal**
  Give upper and lower bounds on $\text{fix}(G)$ in terms of the number n of vertices of G.
- **Intuitively**
  What is the number of vertices we can always keep fixed no matter what planar graph on n vertices we are given and how "bad" the drawing of it is?

Previously known lower bounds

- Paths and cycles (Pach and Tardos 2002):

$$\Omega(\sqrt{n})$$

- Trees (Goaoc et al. 2007):

$$\Omega(\sqrt{n})$$

- General planar graphs (Goaoc et al. 2007, Verbitsky 2007):

$$3$$

Previously known upper bounds

- Cycles (Pach and Tardos 2002):

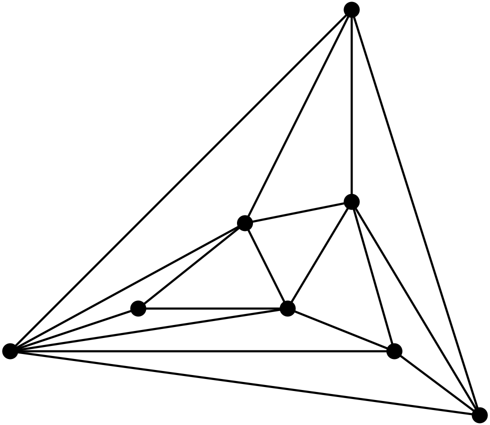
$$O((n \log n)^{2/3})$$

- General planar graphs (Goaoc et al. 2007)

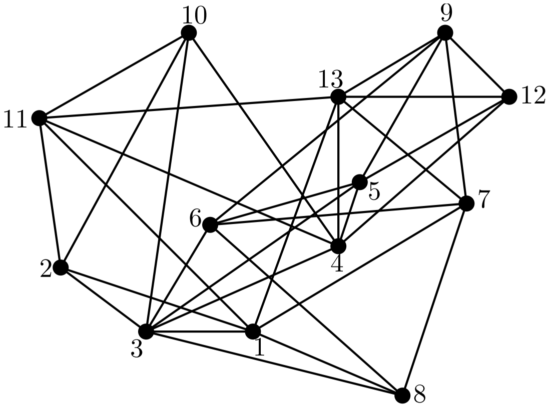
$$O(\sqrt{n})$$

Making our live easy

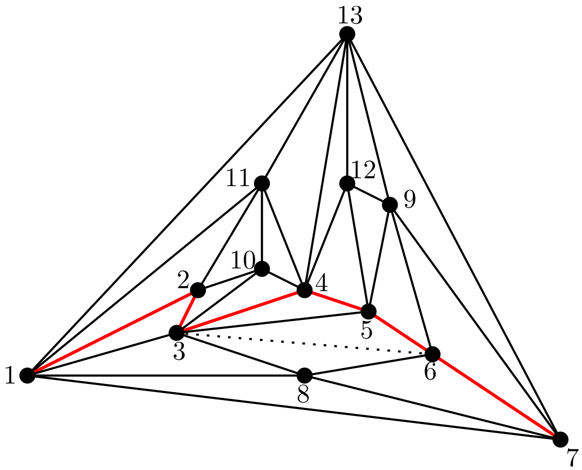
For the lower bound construction we will assume that the given planar graph G is *triangulated*, that is, any additional edge will make G non-planar.



The given drawing

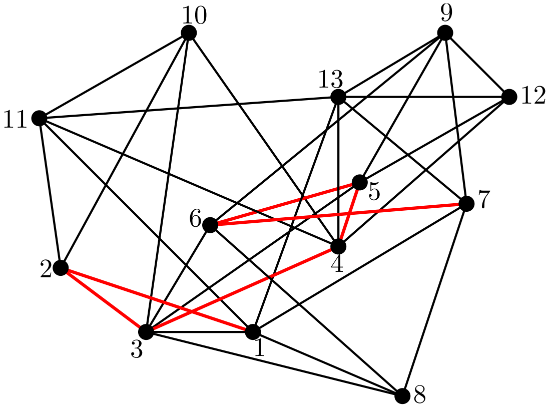


Guiding our construction

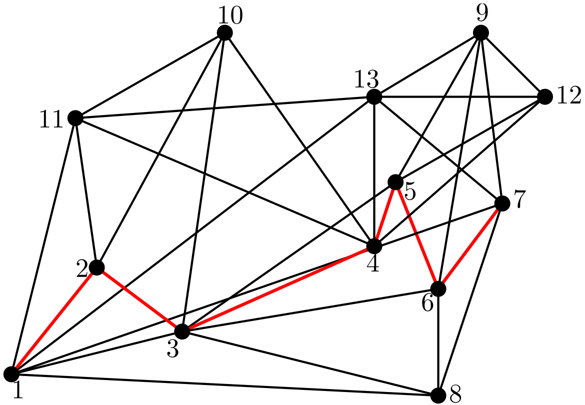


A path with no chords on one side.

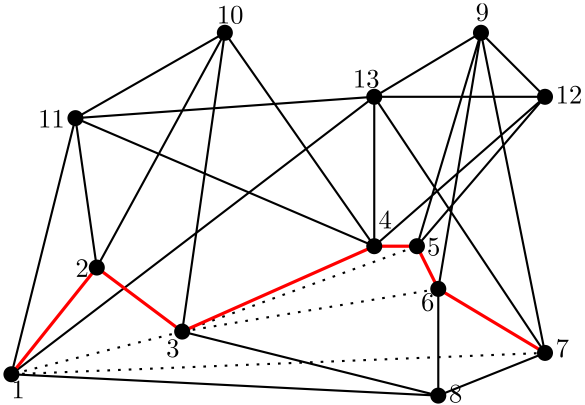
Back to the given drawing



Untangling the path

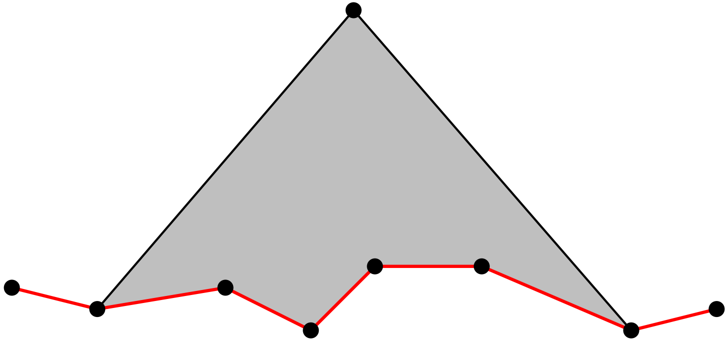


Untangling the chords

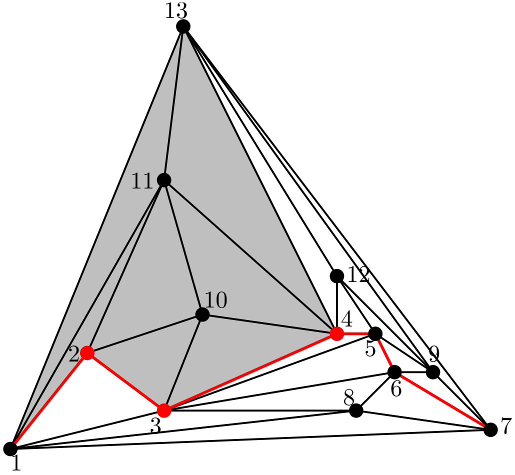


Drawings with star-shaped boundary

(Hong and Nagamochi 2006)



The resulting untangled drawing

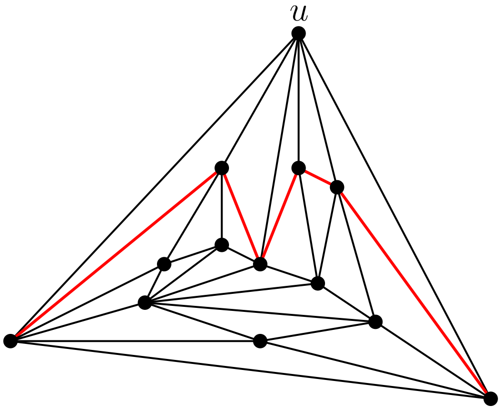


For a path with l vertices we can keep $\Omega(\sqrt{l})$ vertices fixed.

Finding suitable long paths in the given graph

We have

- a vertex u of high degree,



or

- a large diameter (and then using Schnyder Woods).

Our results

Lower bounds:

- General planar graphs:

$$\Omega\left(\sqrt{\frac{\log n}{\log \log n}}\right)$$

- Outerplanar graphs:

$$\Omega(\sqrt{n})$$

Upper bound:

- Outerplanar graphs:

$$O(\sqrt{n})$$

Concluding remarks

Two main results:

- Asymptotically tight lower and upper bounds for the class of outerplanar graphs.
- The path construction outlined in this talk is a main building block in the proof of the recently improved lower bound for general planar graphs (Bose et al. 2007), which yields

$$\Omega(\sqrt[4]{n}).$$