Petri Nets		Reducible Nets

Slicing Petri Nets with an Application to Workflow Verification

Astrid Rakow astrid.rakow@informatik.uni-oldenburg.de

TrustSoft Graduate School, University of Oldenburg, Germany

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Petri Nets	Results	Reducible Nets

1 Outline

Petri Nets

Slicing Algorithm

Results

Reducible Nets

• Workflow Nets

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Image: A image: A

1 Petri Net

$\boldsymbol{\Sigma} = (\boldsymbol{\mathsf{S}}, \boldsymbol{\mathsf{T}}, \boldsymbol{\mathsf{W}}, \boldsymbol{\mathsf{M}_0})$





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1 Petri Net





Image: A math a math

1 Model Checking



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2 Slicing		
Petri Nets	Algorithm	Reducible Nets

Aim: Tackle the State Explosion Problem for Model Checking

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Petri Nets	Algorithm	Reducible Nets
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3 Slicing

Aim: Tackle the State Explosion Problem for Model Checking

Given a marked net $\Sigma,$ find a subnet Σ' such that

(i)
$$\Sigma \models \phi \Rightarrow \Sigma' \models \phi$$
 and $\Sigma' \models \phi \Rightarrow \Sigma \models \phi$
(falsification) (verification)

for a CTL* formula ϕ and

(ii) Σ' has a smaller state space than Σ .

Image: A math a math

4 What do we need to keep



.. if we are interested in whether A FG (s4, 1) holds?

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Image: A math a math

4 What do we need to keep?

Basic Idea:



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Image: A math a math

5 Slicing a Petri Net

```
Given: \Sigma = (S, T, W, M_0) and P \subseteq S a non-empty set.
T'. S<sub>done</sub> := \emptyset;
S' := P:
while (\exists s \in (S' \setminus S_{done}))
    while (\exists t \in (\bullet s \cup s^{\bullet}) : W(s, t) \neq W(t, s)) {
        S' := S' \cup \bullet t
       \mathsf{T}' := \mathsf{T}' \cup \{\mathsf{t}\};
    S_{done} := S_{done} \cup \{s\}:
W' := W|_{T' \cup S'};
M'_{0} := M_{0}|_{S'};
return (S', T', W', M'_0)
```

Image: A matrix and a matrix

Reducible Nets

6 Applying the Algorithm



Image: A math a math

Reducible Nets

7 Applying the Algorithm



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Reducible Nets

8 Applying the Algorithm



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Reducible Nets

9 Applying the Algorithm



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10 Applying the Algorithm



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Image: A math a math

Let ϕ be a **CTL**^{*}_{-x} formula such that ϕ refers to places of the slice Σ' . $\Sigma \models \phi$ fairly w.r.t T' $\Leftrightarrow \Sigma' \models \phi$

Let ϕ be an **LTL** formula such that ϕ refers to places of the slice Σ' .

$\mathbf{\Sigma} \models \phi \qquad \qquad \Rightarrow \quad \mathbf{\Sigma}' \models \phi$

Image: A math a math

12 Fairness

Definition

σ eventually permanently enables $t \in T$ iff

- either σ is finite and t is enabled after firing σ
- or σ is infinite and $\exists i, 0 \leq i : \forall j, i \leq j : M_j[t\rangle$



Image: A math a math

Definition

A maximal σ is fair w.r.t T' iff

- \blacktriangleright either σ is finite
- or σ is infinite and if there is a t \in T' it eventually permanently enables, it then fires infinitely often some $t' \in T'$.



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13 Reducible Nets





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14 Evaluation

Benchmark set of [Corbett96]

- ▶ For each Petri net a slice for each place is built.
- ▶ A slice is nice if it has 10% to 85% of the states.
- ▶ 6/23 with nice slices
- 67 % of places covered in average

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14 Workflows

- A workflow defines the order of **tasks** of a **process**
- Petri nets can represent workflows very intuitively
- Workflow analysis helps to make a process effective and efficient
- \blacktriangleright A P/T net represents the possible behaviour of a workflow

Image: A math a math

15 A Workflow Example



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16 **Conclusions**

- linear-time algorithm
- Petri net is reduced
- state space is reduced

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Thanks for listening !

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References

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